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# Comments on geometric and universal open string tachyons near fivebranes

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Abstract: In a recent work [1], Sen studied unstable D-branes in NS5-brane backgrounds and argued that in the strong curvature regime the universal open string tachyon (on D-branes of the wrong dimensionality) and the geometric tachyon (on D-branes that are BPS in flat space but not in this background) may become equivalent. We study in this note an example of a non-BPS suspended D-brane vs. a BPS D-brane at equal distance between two fivebranes. We use boundary worldsheet CFT methods to show that these two unstable branes are identical.

Keywords: D-branes, Conformal Field Models in String Theory, Tachyon Condensation.

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## Contents



## 1. Introduction

Non-BPS brane configurations, e.g. superstring D-branes with the wrong dimensionality, have attracted lot of interest in recent years due to their relevance in supersymmetry breaking and cosmology (see [2] and references therein).

In [3] Kutasov studied BPS D-branes near an NS5-brane stack with a transverse circle. If the D-brane and the NS5-branes are at antipodal points, the D-brane sits at a maximum of its effective potential as it will tend to fall onto the fivebranes. Its open string spectrum has a *geometric tachyon*, even though far from the NS5-branes the D-brane is BPS. Interestingly, the properties of this geometrical tachyon are closely related to those of the *universal tachyon* on non-BPS branes.

Recently Sen considered further configurations of BPS and non-BPS D-branes near NS5-branes [1]. He proposed that, in the limit where the number of fivebranes is small (i.e. strong curvature in target space) one can identify the geometric tachyon on some BPS branes with the universal tachyon on other non-BPS branes. One example involved on the one hand a BPS brane halfway between two NS5-branes (called G-type) and on the other hand a non-BPS brane stretched between two fivebranes (called S-type). However in this regime all $\alpha'$ corrections (perturbative and non-perturbative) play a role.

In this note we provide an exact worldsheet CFT description of this phenomena. We start with a slightly different system, a ring of $k$ NS5-branes with a non-compact transverse space. D-branes in the near-horizon geometry of the fivebranes ring are known [4, 5]. Among them, one has S-type branes stretched between any pair of fivebranes, and G-type branes whose worldvolume fill a disk at the center of the ring.

In the particular case $k = 2$ it reduces to a system closely related to the model studied by Sen – the near-horizon geometry of a pair of NS5-branes. The brane moduli space becomes smaller, leaving one S-type brane stretched between the NS5-branes and one G-type brane which is a fuzzy point halfway between the fivebranes in the transverse space. For these particular branes, the transverse circle is irrelevant. Therefore we expect to find the same phenomena as in Sen's analysis.

Borrowing the technical details from [5] we find that for $k = 2$ the G-type brane and the S-type brane are identified. They have the same one-point function on the disc and their open string spectrum is identical, built on a tachyon with $M^2 = -\frac{1}{2\alpha'}$.

In section 2 we analyze the two sorts of D-branes in the ring geometry. Then in section 3 we focus on the pair of fivebranes, and show that the two unstable branes coincide. We compare finally our results with those of [1]. Some useful material is gathered in two appendices.

## 2. D-branes near a ring of fivebranes

A supersymmetric superstring background is obtained by spreading $k$ parallel NS5-branes on a circle in their non-compact transverse space. Calling $R$ its radius in string units, one can define a double scaling limit of the system [6] ($g_s \to 0$, $R/g_s$ fixed) in order to reach the near-horizon geometry of all fivebranes simultaneously. Remarkably the ring geometry is an exactly solvable worldsheet CFT [7, 8]. Its transverse part is T-dual to $[\mathrm{SU}(2)_k/\,\mathrm{U}(1) \times \mathrm{SL}(2,\mathbb{R})_k/\,\mathrm{U}(1)]/\mathbb{Z}_k$.

D-branes in this background have been constructed in [4, 5]. First the suspended D-branes are made, in the T-dual background, of a D0-brane of the cigar $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ sitting at the tip and a D2-brane of $\mathrm{SU}(2)/\,\mathrm{U}(1)$ with the shape of a disk. While the former carries no label, the latter has one parameter $\hat{\jmath} = 0, 1/2, \ldots k/2 - 1$ giving its length in the fivebranes geometry. For the flat $\mathbb{R}^{5,1}$ part of the background we choose NNNNDD boundary conditions in type IIB, giving a non-BPS D4-brane stretched between two fivebranes, i.e. an S-type brane (see the right picture of fig 1). The annulus amplitude for open strings between two identical S-type branes reads:

$$\mathcal{A}^{\mathrm{s}}_{\hat{\jmath}\hat{\jmath}} = \sum_{a,\upsilon_i \in \mathbb{Z}_2} (-)^a \int \frac{\mathrm{d}t}{2t} \frac{\Theta_{a+2\upsilon_1,2}\,\Theta_{a+2\upsilon_2,2}}{(8\pi^2\alpha' t)^2 \eta^6} \times \\ \times \sum_{\ell=0}^{\min(2\hat{\jmath},k-2\hat{\jmath})} \sum_{m \in \mathbb{Z}_{2k}} C^{\ell\,(a+2\upsilon_3)}_m(it)\, Ch^{(a+2\upsilon_4)}_{\mathbb{I}}\left(\frac{m-a}{2}; it\right) . \quad (2.1)$$

The supersymmetric $\mathrm{SU}(2)/\,\mathrm{U}(1)$ characters $C^{\ell\,(s)}_m$ and $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ extended identity character $Ch^{(s)}_{\mathbb{I}}(r)$ can be found in app. A. One has a *universal* open string tachyon for $\ell = m = \upsilon_i = 0$ in the NS sector ($a = 0$), of mass squared $M^2 = -\frac{1}{2\alpha'}$.

A second type of brane of interest is made in the T-dual of a D0-brane of $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ and a D1-brane of $\mathrm{SU}(2)/\,\mathrm{U}(1)$, with similar boundary conditions for $\mathbb{R}^{5,1}$. In the $\mathbb{Z}_k$ orbifold theory these boundary conditions are not compatible hence the generalized GSO projection does not act in the open string sector. It corresponds in the fivebranes geometry to a disk at the center of the ring of radius $R \sin \frac{\pi}{k}(2\hat{\jmath}+1)$ [5], i.e. a G-type brane (of nonzero radial

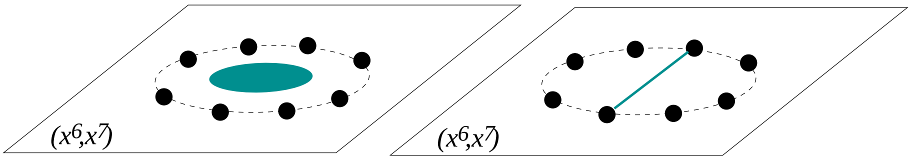


**Figure 1:** G-type (left) and S-type (right) D-branes inside a ring of NS5-branes

extension), see left picture of figure 1. The annulus amplitude for identical branes is:

$$\mathcal{A}^{\mathrm{g}}_{\hat{\jmath}\hat{\jmath}} = \frac{1}{2}\sum_{a,b,\upsilon_i\in\mathbb{Z}_2}(-)^{a+b(1+\sum_i \upsilon_i)}\int \frac{\mathrm{d}t}{2t}\frac{\Theta_{a+2\upsilon_1,2}\Theta_{a+2\upsilon_2,2}}{(8\pi^2\alpha' t)^2\eta^6}\times$$
$$\times \sum_{\ell=0}^{\min(2\hat{\jmath},k-2\hat{\jmath})}\sum_{m\in\mathbb{Z}_{2k}} C^{\ell\,(a+2\upsilon_3)}_m(it)\sum_{r\in\mathbb{Z}_k} Ch^{(a+2\upsilon_4)}_{\mathbb{I}}(r;it)\,. \quad (2.2)$$

The $\frac{1}{2}$ factor comes with the fermionic GSO projection as this brane has the correct dimensionality (odd in type IIB). However states with $r \neq (m-a)/2$ break space-time supersymmetry. The spectrum includes a geometric tachyon (not projected out by the fermionic GSO) for $\ell = r = 0$, $\upsilon_3 = 1$ and $m = 2$ of mass squared $M^2 = -\frac{1}{k\alpha'}$ [4].

## 3. D-branes near a pair of fivebranes

We now specialize the discussion to $k = 2$, i.e. a pair of fivebranes. At first glance we see that the tachyon masses coincide: $M^2 = -\frac{1}{2\alpha'}$.

Let us look in more detail at annulus amplitude for both types of branes. At level $k = 2$ the super-coset $SU(2)_k/U(1)$ simplifies dramatically as it contains only the identity field. From the defining relation of coset characters

$$\chi^j\Theta_{s,2} = \sum_{m\in\mathbb{Z}_{2k}} C^{j\,(s)}_m\Theta_{m,k}\,,$$

one sees that for $k = 2$ one gets the constraint $m = s$ (in addition to $j = 0$). For an S-type brane, the annulus amplitude, eq. (2.1), becomes:

$$\mathcal{A}^{\mathrm{s}}_{k=2} = \sum_{a,\upsilon_i\in\mathbb{Z}_2}(-)^a\int\frac{\mathrm{d}t}{2t}\frac{\Theta_{a+2\upsilon_1,2}\Theta_{a+2\upsilon_2,2}}{(8\pi^2\alpha' t)^2\eta^6}Ch^{(a+2\upsilon_4)}_{\mathbb{I}}(\upsilon_3)\,, \quad (3.1)$$

where the tachyon occurs for $\upsilon_i = 0$, $\forall i$ in the NS sector.

For the G-type brane annulus amplitude the $m = s$ constraint has no effect on the remaining degrees of freedom, therefore one can simply replace in eq. (2.2) the sum of $SU(2)/U(1)$ characters $\sum_m C^{\ell(s)}_m$ by one:

$$\mathcal{A}^{\mathrm{g}}_{k=2} = \frac{1}{2}\sum_{a,b,\upsilon_i\in\mathbb{Z}_2}(-)^{a+b(1+\sum_{i=1}^{4}\upsilon_i)}\int\frac{\mathrm{d}t}{2t}\frac{\Theta_{a+2\upsilon_1,2}\Theta_{a+2\upsilon_2,2}}{(8\pi^2\alpha' t)^2\eta^6}\sum_{r\in\mathbb{Z}_2}Ch^{(a+2\upsilon_4)}_{\mathbb{I}}(r)\,. \quad (3.2)$$



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However, the fermionic $\mathbb{Z}_2$ label $\upsilon_3$ of $\mathrm{SU}(2)/\,\mathrm{U}(1)$ still appears in the phase of the fermionic GSO projection. It gives an overall factor in the annulus amplitude

$$\frac{1}{2}\sum_{\upsilon_3=0}^{1}(-)^{\upsilon_3\, b} = \delta_{b,0 \mod 2}\,. \tag{3.3}$$

Therefore only the $b = 0$ sector (i.e. the untwisted NS and R sectors) survives for $k = 2$. It shows that the annulus amplitudes (3.1), (3.2) are indeed identical once we identify $\upsilon_3$ in eq. (3.1) with $r$ in eq. (3.2).

## 4. Discussion

In this note we compared compact G-type branes (would-be BPS-branes with a geometric tachyon due to the background) and S-type branes (stretched non-BPS branes) inside a ring of fivebranes, using boundary worldsheet CFT. In the limit where the ring consists in two fivebranes, we have shown that the S-type and G-type branes give the same annulus amplitude.

Compared to the setup of [3, 1] the space transverse to the fivebranes is not compactified on a circle. For this particular sector of D-branes that are either stretched or halfway between a pair of fivebranes it is irrelevant. However the transverse circle should be taken into account if one includes also U-type branes wrapping the circle in the discussion.

Firstly, the matching of one-loop open string amplitudes implies that both branes have the same open string tachyon, supporting the conjecture made in [1]. The decay of this tachyon has been studied in [9] using CFT methods. It was found that, as in flat space, the outcome of the process is a dust of very massive closed strings.

Secondly, because the whole annulus amplitudes are identical, by channel duality one finds that the brane couplings to closed strings are the same up to a phase. It can be checked at the level of the boundary states that the couplings are actually identical, see app. B. In particular having identical couplings to the graviton means that the G-type and the S-type branes have the same tension.

Similar correspondences between universal and geometric open string tachyons near a pair of NS5-branes exist for non-compact branes. For instance, the non-BPS "D-rays" of [5], identified in the large $k$ limit as a pair of semi-infinite D1-branes ending on the fivebranes from outside the ring, and the second class of G-type D4-branes (section 5.2 in [5]) with $\hat{\jmath} = 0$, filling the $x^{8,9}$ plane at $x^6 = x^7 = 0$ (the NS5-branes are spread in the $x^{6,7}$ plane), become equivalent for $k = 2$. It would be interesting to investigate these aspects further.

These results illustrate how space-time geometry is modified at large curvatures in string theory, whenever all $\alpha'$ corrections are taken into account.

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## A. $\mathcal{N}=2$ characters

**Free fermions.** In order to write free-fermion characters we use $\vartheta\left[\begin{smallmatrix}a\\b\end{smallmatrix}\right](\tau,\nu) = \sum_{n\in\mathbb{Z}} q^{\frac{1}{2}(n+\frac{a}{2})^2} e^{2i\pi(n+\frac{a}{2})(\nu+\frac{b}{2})}$. It is convenient to split the R and NS sectors according to the fermion number mod 2:

$$\begin{array}{ll} \frac{1}{2\eta}\left\{\vartheta\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]-\vartheta\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]\right\} = \frac{\Theta_{0,2}}{\eta} & \frac{1}{2\eta}\left\{\vartheta\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]+\vartheta\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]\right\} = \frac{\Theta_{2,2}}{\eta} \\ \frac{1}{2\eta}\left\{\vartheta\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]-i\vartheta\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]\right\} = \frac{\Theta_{1,2}}{\eta} & \frac{1}{2\eta}\left\{\vartheta\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]+i\vartheta\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]\right\} = \frac{\Theta_{3,2}}{\eta} \end{array} \tag{A.1}$$

in terms of the theta functions of $\widehat{\mathfrak{su}}(2)$ at level 2: $\Theta_{m,k}(\tau,\nu) = \sum_{n\in\mathbb{Z}} q^{k\left(n+\frac{m}{2k}\right)^2} e^{2i\pi\nu k\left(n+\frac{m}{2k}\right)}$. The modular transformation property of these $\mathrm{U}(1)_2$ characters is then:

$$\frac{\Theta_{s,2}(-1/\tau,\nu/\tau)}{\eta(-1/\tau)} = \frac{1}{2}\, e^{i\pi\nu^2/\tau} \sum_{s'\in\mathbb{Z}_4} e^{-\frac{i\pi ss'}{2}} \frac{\Theta_{s',2}(\tau,\nu)}{\eta(\tau)}. \tag{A.2}$$

**$\mathcal{N}=2$ minimal models.** The characters of the $\mathcal{N}=2$ minimal model, i.e. the supersymmetric gauged WZW model $\mathrm{SU}(2)_k/\mathrm{U}(1)$, are determined implicitly through the identity:

$$\sum_{m\in\mathbb{Z}_{2k}} \mathcal{C}^{j\,(s)}_{m}\Theta_{m,k} = \chi^j\Theta_{s,2}\,, \tag{A.3}$$

where $\chi^j$ is a character of SU(2) at level $k-2$. They are labeled by $(j,m,s)$, corresponding to primaries of the coset $[\mathrm{SU}(2)_{k-2}\times\mathrm{U}(1)_2]/\mathrm{U}(1)_k$. The following identifications apply:

$$(j,m,s)\sim(j,m+2k,s)\sim(j,m,s+4)\sim(k/2-j-1,m+k,s+2) \tag{A.4}$$

as the selection rule $2j+m+s=0 \mod 2$. The weights of the primaries states are:

$$\begin{array}{lll} h = \frac{j(j+1)}{k}-\frac{n^2}{4k}+\frac{s^2}{8} & \text{for} & -2j\leqslant n-s\leqslant 2j \\ h = \frac{j(j+1)}{k}-\frac{n^2}{4k}+\frac{s^2}{8}+\frac{n-s-2j}{2} & \text{for} & 2j\leqslant n-s\leqslant 2k-2j-4 \end{array} \tag{A.5}$$

We have the following modular S-matrix for these characters:

$$S^{j\,m\,s}_{\quad j'\,m'\,s'} = \frac{1}{2k}\sin\pi\frac{(1+2j)(1+2j')}{k}\; e^{i\pi\frac{mm'}{k}}\; e^{-i\pi ss'/2}. \tag{A.6}$$

**Supersymmetric $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$.** The characters of the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ super-coset at level $k$ come in different categories corresponding to irreducible unitary representations of $\mathrm{SL}(2,\mathbb{R})$. The *continuous representations* correspond to $j=1/2+iP$, $P\in\mathbb{R}^+$. Their characters are denoted by $ch_c(p,m)\left[\begin{smallmatrix}a\\b\end{smallmatrix}\right]$, where the $\mathrm{U}(1)_R$ charge of the primary is $Q=2m/k$. The *discrete representations*, of characters $ch_d(j,r)\left[\begin{smallmatrix}a\\b\end{smallmatrix}\right]$, have $1/2<j<(k+1)/2$, with $\mathrm{U}(1)_R$ charge $Q=2(j+r+a/2)/k$, $r\in\mathbb{Z}$. The *identity representation* primaries have $\mathrm{U}(1)_R$ charge $Q=(2r+a)/k$, $r\in\mathbb{Z}$, and the characters read :

$$ch_{\mathbb{I}}(r;\tau,\nu)\begin{bmatrix}a\\b\end{bmatrix} = \frac{(1-q)\; q^{\frac{-1/4+(r+a/2)^2}{k}} e^{2i\pi\nu\frac{2r+a}{k}}}{\left(1+(-)^b\, e^{2i\pi\nu}q^{1/2+r+a/2}\right)\left(1+(-)^b\, e^{-2i\pi\nu}q^{1/2-r-a/2}\right)} \frac{\vartheta\left[\begin{smallmatrix}a\\b\end{smallmatrix}\right](\tau,\nu)}{\eta^3(\tau)}. \tag{A.7}$$

One can define characters labeled by a quantum number $s\in\mathbb{Z}_4$, following the method we used for free fermions. The spectrum of NS primaries for the identity representation is as follows. The identity operator is $|0\rangle_{\rm NS} \otimes |r=0\rangle_{\rm SL(2,R)}$, in the sector $s=0$. The other primaries, in the sector $s=2$, have weights $h = \frac{r^2}{k} + |r| - \frac{1}{2}$.

*Extended characters* are defined for $k$ integer by summing over $k$ units of spectral flow. The extended identity characters are:

$$Ch_{\mathbb{I}}^{(s)}(r;\tau) = \sum_{w\in\mathbb{Z}} ch_{\mathbb{I}}^{(s)}(r+kw;\tau) \quad \text{with} \qquad r \in \mathbb{Z}_k \tag{A.8}$$

Their modular transformation involve only a discrete set of $\mathcal{N}=2$ charges:

$$\begin{aligned}
Ch_{\mathbb{I}}^{(a+2\upsilon)}(r;-1/\tau) &= \frac{1}{k}\sum_{a',\upsilon'\in\mathbb{Z}_2} e^{-\frac{i\pi}{2}(a+2\upsilon)(a'+2\upsilon')} \times \\
&\times \left[\int_0^\infty \mathrm{d}P' \sum_{m'\in\mathbb{Z}_{2k}} e^{-\frac{2i\pi}{k}(r+\frac{a}{2})m'} \frac{\sinh 2\pi P' \sinh\frac{2\pi P'}{k}}{\cosh 2\pi P' + \cos\pi(m'-a')} Ch_c^{(a'+2\upsilon')}(P',\frac{m'}{2};\tau)\right. \\
&\left. + \sum_{2j'-1=1}^{k-2}\sum_{r'\in\mathbb{Z}_k} \sin\frac{\pi(1-2j')}{k}\, e^{-\frac{4i\pi}{k}(j'+r'+\frac{a'}{2})(r+\frac{a}{2})} Ch_d^{(a'+2\upsilon')}(j',r';\tau)\right]
\end{aligned} \tag{A.9}$$

## B. Boundary states

**S-type brane boundary state.** We consider the boundary state for a type IIB S-type D4-brane in $\mathbb{R}^{5,1}\times[\mathrm{SU}(2)_k/\mathrm{U}(1)\times \mathrm{SL}(2,\mathbb{R})_k/\mathrm{U}(1)]/\mathbb{Z}_k$. The coefficients of the Ishibashi states are given by the one-point functions (4.7) of [5][1] (see also [10]), keeping only couplings to the NS-NS sector with an extra $\sqrt{2}$ factor to satisfy the Cardy condition (as non-BPS branes in flat space). Its labels $(\hat{\jmath},\hat{m})$ (with $0\leqslant 2\hat{\jmath}\leqslant k-2$ and $\hat{m}\in\mathbb{Z}_{2k}$) give the position of the two fivebranes on which the D-brane ends; we choose $\hat{\jmath}=0$ (i.e. shortest length). As in flat space the brane has fermionic labels $\{\hat{s}_i\in\mathbb{Z}_4, i=1,\ldots,4\}$, and its position is $(\hat{x}^4,\hat{x}^5)$. The couplings to $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ continuous representations are (the discrete ones follow from analyticity):

$$\begin{aligned}
|\mathrm{S}\,(\hat{x}^\mu,\hat{\jmath}=0,\,\hat{m},\,\{\hat{s}_i\})\,\rangle_k &= \mathcal{T}\frac{\sqrt{2}}{k}\int\frac{d^2p}{(2\pi)^2}\, e^{i(p_4\hat{x}^4+p_5\hat{x}^5)} \sum_{\upsilon_i\in\mathbb{Z}_2}\frac{1}{2}\sum_{b=0}^{1}(-)^{b(1+\sum_i\upsilon_i)} \times \\
&\times\; e^{i\pi\sum_{i=1}^4\upsilon_i\hat{s}_i}|p_i,s_1=-\bar{s}_1=2\upsilon_1,s_2=-\bar{s}_2=2\upsilon_2\rangle\rangle_{\rm flat} \\
\otimes \sum_{2j'=0}^{k-2}\sum_{m\in\mathbb{Z}_{2k}}\delta_{2j'+m,0 \mod 2}\, & e^{-\frac{i\pi}{k}m\hat{m}}\sqrt{\sin\frac{\pi}{k}(2j'+1)}|j',m,-m,s_3=\bar{s}_3=2\upsilon_3\rangle\rangle_{\mathfrak{su}(2)/\mathfrak{u}(1)} \\
\otimes \int_0^\infty dP\,\nu_k^{-iP}\sum_{w\in\mathbb{Z}} & \frac{\Gamma(\frac{1}{2}+iP+\frac{m}{2}-\upsilon_4+kw)\,\Gamma(\frac{1}{2}+iP-\frac{m}{2}+\upsilon_4-kw)}{\Gamma(2iP)\,\Gamma(1+2iP/k)}\times \\
&\times\;|j=1/2+iP,m/2+kw,-m/2-kw,s_4=-\bar{s}_4=2\upsilon_4\rangle\rangle_{\mathfrak{sl}(2)/\mathfrak{u}(1)}
\end{aligned}\tag{B.1}$$

[1]Compared to [5] the brane has a different dimensionality. The fivebranes are stretched along $x^{0,\ldots,5}$ and distributed in the $x^{6,7}$ plane, the D4-brane is stretched along $x^{0,1,2,3}$ and one direction in the $x^{6,7}$ plane.

with the normalization $\mathcal{T} = (\frac{2}{\alpha'})^{\frac{1}{2}}/8\pi$ from $\mathbb{R}^{5,1}$. The sum over $b = 0, 1$ implements the closed string fermionic GSO projection. We denote by $|\star\rangle\rangle$ the Ishibashi states for flat space with NNDD boundary conditions, supersymmetric SU(2)/ U(1) and SL(2, $\mathbb{R}$)/ U(1) with B-type boundary conditions respectively.

Let us now choose $k = 2$. As we saw in the bulk of the paper, for the SU(2)/ U(1) coset we have only the identity, i.e. $j' = 0$ and $m = s_3 = 2\upsilon_3$. It gives the boundary state:

$$
\begin{aligned}
|\mathrm{S}\,(\hat{x}^\mu, \hat{m}, \{\hat{s}_i\})\,\rangle_2 = \mathcal{T}\frac{1}{\sqrt{2}} \int \frac{d^2p}{(2\pi)^2}\; e^{i(p_4\hat{x}^4+p_5\hat{x}^5)} \sum_{\upsilon_i\in\mathbb{Z}_2} \frac{1}{2}\sum_{b=0}^{1}(-)^{b(1+\sum_i \upsilon_i)} e^{i\pi\sum_{i=1}^4 \upsilon_i\hat{s}_i} \times \\
\times\; |p_i, s_1 = -\bar{s}_1 = 2\upsilon_1, s_2 = -\bar{s}_2 = 2\upsilon_2\rangle\rangle_{\text{flat}} \;\otimes\; e^{-i\pi\upsilon_3\hat{m}}\; |0\rangle\rangle_{\mathfrak{su}(2)/\mathfrak{u}(1)} \\
\otimes \int_0^\infty dP\; \nu_2^{-iP} \sum_{w\in\mathbb{Z}} \frac{\Gamma(\frac{1}{2}+iP+\upsilon_3-\upsilon_4+2w)\,\Gamma(\frac{1}{2}+iP-\upsilon_3+\upsilon_4-2w)}{\Gamma(2iP)\,\Gamma(1+iP)} \times \\
\times\; |j = 1/2+iP, \upsilon_3+2w, -\upsilon_3-2w, s_4 = -\bar{s}_4 = 2\upsilon_4\rangle\rangle_{\mathfrak{sl}(2)/\mathfrak{u}(1)} \quad \text{(B.2)}
\end{aligned}
$$

**G-type brane boundary state.** We consider the G-type D5-brane, still in type IIB. The brane has the shape of a disc in the $x^{6,7}$ plane, whose radius is parameterized by $\hat{\jmath}$. We choose $\hat{\jmath} = 0$, i.e. a "fuzzy" D3-brane at the center of the ring. The coefficients of the Ishibashi states are obtained from the one-point function (5.2) of [5]. One gets the G-type D3-brane boundary state as [4, 5]:[2]

$$
\begin{aligned}
|\mathrm{G}\,(\hat{x}^\mu, \hat{\jmath}=0, \eta, \{\hat{s}_i\})\,\rangle_k = \mathcal{T}\frac{1}{\sqrt{k}} \int \frac{d^2p}{(2\pi)^2}\; e^{i(p_4\hat{x}^4+p_5\hat{x}^5)} \sum_{\upsilon_i\in\mathbb{Z}_2} \frac{1}{2}\sum_{a,b=0}^{1}(-)^{a+b(1+\sum_i \upsilon_i)} \times \\
\times\; e^{\frac{i\pi}{2}\sum_{i=1}^4(a+2\upsilon_i)\hat{s}_i}\; |p_i, s_1 = -\bar{s}_1 = a+2\upsilon_1, s_2 = -\bar{s}_2 = a+2\upsilon_2\rangle\rangle_{\text{flat}} \\
\otimes \sum_{2j'+1=1}^{k-1} \sum_{\epsilon\in\mathbb{Z}_2} \eta^\epsilon \sqrt{\sin\frac{\pi}{k}(2j'+1)}\; \delta_{2j'+k\epsilon+a,0 \mod 2} |j', 0, 0, s_3 = -\bar{s}_3 = a+2(\upsilon_3+\epsilon)\rangle\rangle_{\mathfrak{su}(2)/\mathfrak{u}(1)} \\
\otimes \int_0^\infty dP\; \nu_k^{-iP} \sum_{w\in\mathbb{Z}} \frac{\Gamma(\frac{1}{2}+iP+\frac{k\epsilon}{2}-\upsilon_4+kw)\,\Gamma(\frac{1}{2}+iP-\frac{k\epsilon}{2}+\upsilon_4-kw)}{\Gamma(2iP)\,\Gamma(1+2iP/k)} \times \\
\times\; |j = 1/2+iP, k(w+\epsilon/2), -k(w+\epsilon/2), s_4 = -\bar{s}_4 = a+2\upsilon_4\rangle\rangle_{\mathfrak{sl}(2)/\mathfrak{u}(1)} \quad \text{(B.3)}
\end{aligned}
$$

where now the SU(2)/ U(1) Ishibashi states correspond to A-type boundary conditions. The label $\eta = \pm 1$ is a $\mathbb{Z}_2$-valued Wilson line [11].

Let us consider the case $k = 2$. The states that survive have $j' = 0$ and $a+2(\upsilon_3+\epsilon) = 0$, hence only the NS-NS sector (i.e. $a = 0$) together with the constraint $\epsilon = -\upsilon_3 \mod 2$. One gets the same boundary state as for the S-type brane, eq. (B.2), once we identify the brane labels as $\eta = e^{i\pi\hat{m}}$.

## References


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